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Title: **METHOD AND SYSTEM FOR CONCURRENT ESTIMATION OF FREQUENCY OFFSET AND MODULATION INDEX**

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METHOD AND SYSTEM FOR CONCURRENT ESTIMATION OF FREQUENCY OFFSET AND MODULATION INDEX

CROSS-REFERENCE TO RELATED APPLICATIONS

This patent application claims the benefit of priority from and incorporates by reference the entire disclosure of U.S. Provisional Patent Application No. 60/392,114, filed on June 27, 2002.

BACKGROUND OF THE INVENTION

Technical Field of the Invention

The present invention relates generally to the field of radio receivers that utilize continuous phase modulation (CPM) and, more particularly, to a method of and system for estimating a modulation index and a carrier frequency offset of a CPM signal.

Description of Related Art

Wireless technologies, such as terrestrial and satellite mobile communications and/or BLUETOOTH systems, may use continuous-phase-modulated (CPM) signals to transmit data. Binary CPM or M-ary CPM may be employed for the wireless transmission of data packets. When data is transmitted using CPM, the modulation index may need to be known in some receiver architectures.

To improve performance, receiver architectures could be employed that require knowledge of the value of the modulation index of the transmitted signal. Due to the use of independent frequency generating circuits in the transmitting and receiving devices, a carrier frequency offset is typically generated. In order to achieve optimal performance, the carrier

frequency offset should be compensated for as much as possible. Therefore, there is a need for a method of and system for estimating the modulation index and the carrier frequency offset of a CPM signal.

SUMMARY OF THE INVENTION

These and other drawbacks are overcome by embodiments of the present invention, which provide a method of and system for concurrent estimation of a modulation index and frequency offset of a CPM signal. An estimator for estimating a modulation index and frequency offset of a received continuous-phase-modulated (CPM) signal includes at least two filters for filtering the received CPM signal, a calculator for calculating an α value and a β value, and a processor for receiving a signal output by each of the at least two filters, the α value, and the β value. The processor is adapted to calculate estimates of the modulation index and frequency offset from the signals received by the processor and the received α value and β value.

A method of estimating a modulation index and frequency offset of a received continuous-phase-modulated (CPM) signal includes filtering the received CPM signal via at least two filters, calculating an α value and a β value, receiving a signal output by each of the at least two filters, the α value, and the β value, and calculating estimates of the modulation index and frequency offset from the received signals and the received α value and β value.

An estimator for estimating a modulation index and frequency offset of a received continuous-phase-modulated (CPM) signal includes a noise whitener for whitening noise of the received CPM signal, at least two filters for filtering the noise-whitened CPM signal, an initializer for processing a training sequence, and a processor for receiving a signal output by each of the at least two filters and the processed training sequence. The processor is adapted

to calculate estimates of the modulation index and frequency offset from the signals received by the processor and the processed training sequence.

A method of estimating a modulation index and frequency offset of a received continuous-phase-modulated (CPM) signal includes whitening noise of the received CPM signal, filtering the noise-whitened CPM signal via at least two filters, processing a training sequence, receiving a signal output by each of the at least two filters and the processed training sequence, and calculating estimates of the modulation index and frequency offset from the received signals and the processed training sequence.

An estimator for estimating a modulation index and frequency offset of a received continuous-phase-modulated (CPM) signal includes at least two filters for filtering the received CPM signal, a noise whitener for whitening noise of a signal output by the at least two filters, an initializer for processing a training sequence, a processor for receiving signals output by the noise whitener and the processed training sequence. The processor is adapted to calculate an estimate of the modulation index and the frequency offset from the received signals and the processed training sequence.

An estimator for estimating a modulation index and frequency offset of a received continuous-phase-modulated (CPM) signal includes a receiver for receiving the CPM signal and a processor for estimating the modulation index and frequency offset according to the following equation $v = (B^T C^{-1} B)^{-1} B^T C^{-1} \phi$. v represents a vector including elements representing scaled versions of estimates of the modulation index and the frequency offset. C represents a noise covariance matrix, B represents a data model matrix, and ϕ is an observation vector that represents a phase of the CPM signal.

Further advantages and specific details of the present invention will become apparent hereinafter from the detailed description given below in conjunction with the following drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

A more complete understanding of exemplary embodiments of the present invention can be achieved by reference to the following Detailed Description of Exemplary Embodiments of the Invention when taken in conjunction with the accompanying Drawings, wherein:

FIGURE 1 is a block diagram that schematically illustrates an estimator in accordance with principles of the present invention;

FIGURE 2 is a block diagram of the estimator of FIGURE 1, including a bias removal component in accordance with principles of the present invention;

FIGURE 3 is a block diagram that schematically illustrates another estimator in accordance with principles of the present invention;

FIGURE 3A is a block diagram that schematically illustrates an estimator in accordance with principles of the present invention;

FIGURE 3B is a block diagram that schematically illustrates an estimator in accordance with principles of the present invention; and

FIGURE 4 is a block diagram that schematically illustrates a noise-whitening estimator in accordance with principles of the present invention.

DETAILED DESCRIPTION OF EXEMPLARY EMBODIMENTS OF THE INVENTION

In accordance with principles of the present invention, several approaches can be taken to estimate the modulation index and carrier frequency offset of a CPM signal. The estimators associated with aspects of the present invention may be divided into two distinct categories, namely estimators that assume white noise and estimators that assume colored noise. Another classification within the two distinct categories may be made based upon whether or not Inter-Symbol Interference (ISI) is assumed to be present in the input signal. When it is assumed that ISI is present, two further subclasses may be introduced based on whether or not the value of a parameter ε is known or unknown. The estimators described hereinafter are based upon the least-squares approach and can be represented by the following equation:

$$\nu = (B^T C^{-1} B)^{-1} B^T C^{-1} \phi \qquad (1)$$

In equation (1) above, ν is a vector that includes elements representing scaled versions of the estimates of the modulation index and the frequency offset. The matrix C represents a noise covariance matrix and the matrix B represents the data model. The last three terms in equation (1) are a filter operation on an observation vector ϕ which is the phase input to the estimator.

A first approach uses an estimator based upon a simple data model, which does not take into account the Inter-Symbol Interference (ISI). The first approach, as shown in detail in FIGURE 1, assumes white noise and no ISI. By assuming that bit timing and frame synchronization are known, an output (i.e., an element of the observation vector) ϕ_k of a 1-bit differential demodulator at an optimal sampling moment can be modeled as:

$$\begin{aligned}\phi_k &= b_k h\pi + 2\pi f T_{sym} + n_k \\ \phi_k &= b_k + y + n_k\end{aligned} \qquad (2)$$

where b_k is a transmitted bit, h is a modulation index, f is an actual frequency offset, T_{sym} is a symbol period, and n_k is a distortion term that includes noise and ISI. By omitting the noise terms n_k, equation (2) can be rewritten in matrix notation as follows:

$$\phi = Bv \text{ in which } \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_N \end{bmatrix}, \; v = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 & 1 \\ b_2 & 1 \\ b_3 & 1 \\ \vdots & 1 \\ b_N & 1 \end{bmatrix} \qquad (3)$$

If $\{b_i\}_{i=1}^{N}$ (i.e., N transmitted bits) are known, equation (3) can be solved by multiplying the observation vector ϕ by the pseudo-inverse of B. Assuming white noise is present, the covariance matrix C is equal to the identity matrix. Therefore, equation (1) may be rewritten as:

$$v = (B^T B)^{-1} B^T \phi \qquad (4)$$

wherein the superscript T denotes the transpose of the matrix B and the superscript -1 denotes the inverse of the resulting matrix shown in parentheses.

Given the above structure of the data matrix B, the following equations prove true:

$$B^T B = \begin{bmatrix} N & S \\ S & N \end{bmatrix} \text{ and } (B^T B)^{-1} = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \qquad (5)$$

with $S = \sum_{k=1}^{N} b_k$, $\alpha = \frac{N}{N^2 - S^2}$ and $\beta = \frac{-S}{N^2 - S^2}$

The value of S directly relates to a weight W of the known bit sequence as follows:

$$S = 2W - N \qquad (6)$$

By implementing a least-squares method, the values of x and y of the vector v can be estimated. The estimates of the modulation index h and the frequency offset f may be

directly derived from the estimates of x and y respectively. The estimates of the values of x and y of the vector v can be obtained by applying two linear operations on the observation vector ϕ and a post-processing step that depends on the weight of the training sequence. As such, equation (4) remains true for v as follows:

$$v = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \text{ with } q = B^T \phi = \begin{bmatrix} \sum_{k=1}^{N} b_k \phi_k \\ \sum_{k=1}^{N} \phi_k \end{bmatrix} \tag{7}$$

wherein q_1 and q_2 are the elements of the vector q, with q_1 representing the output of the first filter (correlation operation) and q_2 representing the output of the second low-pass comb filter. FIGURE 1 illustrates a block diagram of an estimator 100 in accordance with principles of the present invention. The estimator 100 of FIGURE 1 is based on the data matrix shown in equation (3). The estimator 100 implements the operations of equation (7) and, as mentioned above, assumes white noise and no ISI. A received signal ϕ_k (the signal received, mixed down to base-band, and differentially demodulated) is passed through a first finite-impulse-response (FIR) filter 102 to yield q_1. The coefficients for the correlation filter 102 are +1 or −1. The received signal ϕ_k is also passed through a second FIR filter 104 to yield q_2.

As described above, the training sequence, which is a data sequence known at both the receiver and the transmitter, can be used to derive α and β. The calculated α is output to a first multiplier 106A and a fourth multiplier 106D. The derived β is output to a second multiplier 106B and a third multiplier 106C. q_1, which is output by the first FIR filter 102, is multiplied, at the first multiplier 106A, with the derived α. q_1 is also multiplied, at the second multiplier 106B, with the derived β.

q_2, which is output by the second FIR filter 104, is multiplied with the derived β at the third multiplier 106C. q_2, which is output by the second FIR filter 104, is also multiplied with the derived α at the fourth multiplier 106D.

The result output by the first multiplier 106A and the result output by the third multiplier 106C are added at a first adder 108A. The result output by the second multiplier 106B and the result output by the fourth multiplier 106D are added at a second adder 108B. The result output by the first adder 108A is x from equation (2). From equation (2), x can be scaled to yield an estimate of the modulation index h. As shown in equation (2), by multiplying x with $1/\pi$, the modulation index h is produced. Therefore, at multiplier 110A, x is multiplied with $1/\pi$, thereby yielding an estimate of the modulation index h. As is also evident from equation (2), the output of the second adder 108B, y can be multiplied by $1/(2\pi T_{sym})$ at multiplier 110B to produce an estimate of the frequency offset f.

Because of the data model used, the simple estimator 100 might yield biased results. For example, a non-zero mean noise term or correlation between the noise and the desired signal might cause the simple estimator 100 to produce unsatisfactory results. The bias in the estimate of the modulation index typically depends on one or more of the frequency offset, the modulation index, and the value of a signal-to-noise ratio E_b/N_o. Most typically, no significant bias is present for the estimate of the frequency offset.

In the event that E_b/N_o is difficult to estimate, the bias in the modulation-index estimate can be compensated for at a particular value of E_b/N_o. For example, the value of E_b/N_o at which the receiver operates at a bit-error rate (BER) of 10^{-3} can be selected. In addition, because the bias in the modulation-index estimate depends on the modulation index itself, the bias can be compensated for at a typical modulation index value, such as, for example, 0.32.

A post-processing step in accordance with principles of the present invention takes into account the fact that the bias in the estimate of the modulation index h depends approximately quadratically on the estimated frequency offset f in order to compensate for the bias in the modulation index estimate. For a bias-compensated modulation index h_{comp}, the following quadratic equation holds:

$$h_{comp} = h + c_o + c_2 y^2 \qquad (8)$$

The coefficients c_0 and c_2 are chosen via a curve-fitting process in order to minimize the bias. The bias in the modulation-index estimate and the frequency-offset estimate can be derived by simulation. From the simulation results, adequate bias-reduction processes could be derived via curve fitting..

FIGURE 2 illustrates the estimator of FIGURE 1 with additional post-processing to remove bias from the estimate of the modulation index h. The estimator 200 of FIGURE 2 is similar to the estimator 100 of FIGURE 1, except for the implementation of additional components used to introduce the coefficients c_0 and c_2. As noted above, the coefficients c_0 and c_2 are used to remove bias from the estimate of the modulation index h.

As indicated in equation (8), y, which is output by the second adder 108B, is squared by a squaring block 202. An output of the squaring block 202 is multiplied with the value of c_2 at a multiplier 204. An output of the multiplier 204 is added, at an adder 206, to c_0 and to the estimate of the modulation index h. As noted above, the estimate of the modulation index h is output by the multiplier 110A. The adder 206 outputs the bias-compensated modulation index h_{comp}.

The simple estimators 100 and 200 represent relatively computationally-efficient implementations; however, simplification of the data model implemented by the estimators 100 and 200 might not always produce optimal results. Therefore, an estimator based on a

more-complex data model than that used in the estimators 100 and 200 can be utilized in another embodiment of the present invention.

FIGURE 3 is a block diagram that schematically illustrates a more-complex estimator 300 in accordance with principles of the present invention. The more-complex estimator assumes white noise in a manner similar to that of estimators 100 and 200. However, the more-complex estimator 300 assumes that ISI is present in the signal and further that the parameter ε (see data model from equation (9) shown below) is unknown.

In the estimator 300, instead of processing the input signal as modeled by equation (3) and implemented in the estimators 100 and 200, the model on which the estimator 300 is based is a more complicated model, namely equation (12) shown below. To limit the complexity of the estimator 300, a relatively simple ISI model has been assumed. However, other ISI models can be used without departing from principles of the present invention.

In the estimator 300, a linear relationship between a differential output phase θ_k of a transmitter and the input phase ϕ_k of the estimator is assumed. The linear relationship of the differential output phase θ_k and the input phase ϕ_k can be modeled as follows:

$$\phi_k = \varepsilon\theta_{k-1} + (1-2\varepsilon)\theta_k + \varepsilon\theta_{k+1} \qquad (9)$$

Equation (9) shows the relationship between the input phase ϕ_k and the output phase θ_k and allows for ISI by the parameter ε. If no ISI is present, then the parameter ε has a value of zero.

The differential output phase θ_k can be represented by the following equation:

$$\theta_k = \varphi_k - \varphi_{k-1} = b_k h\pi \qquad (10)$$

wherein φ_k is the phase of a transmitted symbol a_k. Combining equations (9) and (10) and adding the impact of the frequency offset f and the distortion term n_k yields:

$$\phi_k = b_k h\pi + (b_{k-1} - 2b_k + b_{k+1})\varepsilon h\pi + 2\pi f T_{sym} + n_k \qquad (11)$$

$$\phi_k = b_k x + c_k y + z + n_k$$

$$\text{with } c_k = (b_{k-1} - 2b_k + b_{k+1})$$

Equation (11) shows that the ISI exhibits a relationship with the foregoing bit and the following bit. However, in more severe ISI cases, the ISI may exhibit a relationship with the previous two bits and the following two bits. If the distortion term n_k is omitted, equation (11) can be written in matrix notation as follows:

$$\phi = Bv \text{ in which } \phi = \begin{bmatrix} \phi_2 \\ \phi_3 \\ \phi_4 \\ \vdots \\ \phi_{N-1} \end{bmatrix}, \ v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} b_2 & c_2 & 1 \\ b_3 & c_3 & 1 \\ b_4 & c_4 & 1 \\ \vdots & \vdots & \vdots \\ b_{N-1} & c_{N-1} & 1 \end{bmatrix} \qquad (12)$$

When the ISI is not neglected, as mentioned above with respect to equation (11), the foregoing bit and the following bit are required, and therefore the index of equation (12) begins with b_2 and ends with b_{N-1}.

If the N-2 transmitted bits $\{b_2 \ldots b_{N-1}\}$ are known, equation (12) can be solved by multiplying the observation vector ϕ with the pseudo-inverse of B, such that $v = (B^T B)^{-1} B^T \phi$, as shown in equation (4) above.

Referring again to FIGURE 3, the estimator 300, which is described mathematically in equations (4), (11), and (12), requires, in addition to the filtering and correlation shown in the estimator 200, another filter, graphically represented as a middle filter 306. The filtering and correlation of filters 302 and 304 operate in a manner similar to filters 102 and 104 of FIGURE 2. The middle filter 306 has N-2 coefficients c_k. For the coefficients c_k, the following holds: $c_k \in \{0, \pm 2, \pm 4\}$.

Variables x and z are manipulated by multipliers 310A and 310B in a manner similar to that shown for x and y in FIGURE 2 to yield the modulation index h and the frequency

offset f. In cases where the ISI is dominated by the transmitter characteristics and the receiver filter chain, the value of the parameter ε from equation (9) may be assumed to be known. The parameter ε is deduced given the overall filter chain in the transmitter and receive parts of the transceiver. Therefore, the estimator 300 may be simplified by assuming, in addition to white noise and ISI, that the parameter ε is known. Due to this fact, the estimator 300 can be simplified and the number of filters utilized reduced as shown in FIGURES 3A and 3B.

In a first simplified implementation, shown in FIGURE 3A, the estimator 300A includes a correlator filter that is slightly more complex because the filter coefficients are no longer +1 or −1 as in the simple estimators 100, 200, and 300. For the first simplified implementation of the estimator 300A, equation (10) is substituted into equation (9), and equation (11) may be rewritten as follows:

$$\begin{aligned} \phi_k &= (\varepsilon b_{k-1} + (1-2\varepsilon)b_k + \varepsilon b_{k+1})h\pi + 2\pi f T_{sym} + n_k \\ \phi_k &= d_k x + y + n_k \end{aligned} \qquad (13)$$

with $d_k = (\varepsilon b_{k-1} + (1-2\varepsilon)b_k + \varepsilon b_{k+1})$, $x = h\pi$ and $y = 2\pi f T_{sym}$

In the implementation shown in equation (13), the value of the parameter ε is assumed to be known. By omitting the distortion term n_k, equation (13) can be rewritten in matrix form as follows:

$$\phi = Bv \text{ in which } \phi = \begin{bmatrix} \phi_2 \\ \phi_3 \\ \phi_4 \\ \vdots \\ \phi_{N-1} \end{bmatrix}, \; v = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} d_2 & 1 \\ d_3 & 1 \\ d_4 & 1 \\ \vdots & \vdots \\ d_{N-1} & 1 \end{bmatrix} \qquad (14)$$

The implementation of the estimator 300A derived from equation (14) requires two filters: 1) a low-pass filter (304) similar to that in the estimators 100 and 200; and 2) a correlation filter (302) that is matched to the channel (i.e., a matched correlator). Therefore, the N-2 filter

coefficients d_k are no longer +1 or −1 but take values of the set {±1, ±(1-2ε), ±(1-4ε)}. As such, the correlation filter is more complex than that of the estimators 100, 200, and 300. The modulation index h and the frequency offset f are calculated in a manner similar to that of FIGURE 3. The vector-matrix multiplier 308 outputs variables x and y, which are in turn manipulated by multipliers 310A and 310B to form the estimates of the modulation index h and frequency offset f.

The first implementation of the estimator 300A requires the matched correlator. The matched correlator has increased computational complexity; therefore, a second implementation of the estimator 300 with reduced computation complexity, is described below.

Referring now to FIGURE 3B, the second implementation of the estimator of FIGURE 3 is illustrated. The second implementation is not as complex as the first implementation; however, the second implementation is more complex than the estimators 100 or 200. In a manner comparable to that of the estimator 200, the second implementation includes a post-processing step that needs to be executed only once.

The description of the estimator 300 of FIGURE 3 from equation (12) and (3) produces the following equation:

$$v = (B^T B)^{-1} B^T \phi = (B^T B)^{-1} w \quad \text{with } w = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{15}$$

p, q, and r are the outputs of the three filters 302, 306, and 304 as shown in the estimator 300. If the value of ε is known, then q, which is the output from the middle filter 306, is not required. As shown in FIGURE 3B, the middle filter 306 has been eliminated. With P_{ij} representing the element of the matrix $B^T B$ on row i and column j and because $B^T B$ is symmetric, the following equation results:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{23} \\ P_{13} & P_{23} & P_{33} \end{bmatrix} \begin{bmatrix} h\pi \\ \varepsilon h\pi \\ 2\pi f T_{sym} \end{bmatrix} \tag{16}$$

In accordance with equation (16), the following equations prove true:

$$h\pi = \frac{P_{33}p - P_{13}r}{(P_{11}P_{33} - P_{13}^{2}) - (P_{13}P_{23} - P_{12}P_{33})\varepsilon} \text{ and} \tag{17}$$

$$2\pi f T_{sym} = \frac{-P_{13}p + P_{11}r + (P_{12}P_{13} - P_{11}P_{23})\varepsilon h\pi}{P_{11}P_{33} - P_{13}^{2}} \tag{18}$$

Equations (17) and (18) can be manipulated via a processor 320 to yield the estimate of the modulation index h and the frequency offset estimate f. The second implementation of the estimator 300B provides significant advantages over the estimators 100 and 200 when ISI is present.

A third class of estimators includes noise whitening to further improve the performance of the modulation index h and the frequency offset f estimators. A specific configuration of a noise-whitening estimator is given in FIGURE 4 which is a block diagram of a noise whitening estimator. Due to differential demodulation preceding the estimation, the distortion term n_k no longer exhibits typical white noise characteristics. Once the covariance of the matrix of the noise is known, the estimation process can be improved.

The one-sided autocorrelation function R_{nn} of the noise process n_k can be approximated by the following:

$$R_{nn} = [1 \quad -0.5] \tag{19}$$

Taking into account the noise covariance matrix C, which may be directly deduced from the one-sided autocorrelation function R_{nn}, equation (15) may be replaced by equation (1). The noise whitening is performed by multiplication of the inverse C^{-1} of the noise covariance matrix C. The matrix multiplication of $B^T C^{-1}$ with the observation vector ϕ can be

implemented in two ways. In a first option, which is used in the estimator of FIGURE 4, this operation is implemented by applying n (n being equal to the number of columns of matrix B) filters in parallel (multiplying with B^T). In a second option, the operation above is instead performed by two subsequent filter operations, where the first filter operates on the observation vector ϕ to whiten the noise present in that vector, i.e. by multiplication of C^{-1}. Then the output of this filter is fed to the n filters in parallel (n being equal to the number of columns in matrix B), i.e. multiplication by B^T. Both options are functionally the same. In the second option, the whitening of the noise is performed explicitly, while in the first option, the noise whitening is implicitly performed. The estimator described by equation (1) does not restrict the values of the filter coefficients to +1 or −1, thereby increasing complexity over both the estimators 100, 200, and 300. The estimator described by equation (1) is an improved noise-whitening estimator that outperforms the estimators 100, 200, and 300 at the cost of increased complexity.

To reduce complexity of the estimator described by equation (1), a first option is to quantize the inverse of the noise covariance matrix C. Although the complexity might be reduced, the quantization introduces a performance loss in the estimator. A second option is to adapt the structure of the inverse of the noise covariance matrix C. A finite-impulse-response (FIR) filter may be utilized to whiten the noise. Due to the differential demodulated estimator input signal the noise has a high-pass characteristic. Approximated whitening can be achieved by passing the signal through a low-pass filter. One attractive solution would be to use a K-tap comb filter. K may be chosen such that a good balance is obtained between performance loss (compared to ideal whitening) and complexity reduction. A third option would be to implement the approximated whitening operation by means of a low-pass infinite impulse response (IIR) filter.

Any of the three options discussed above can be utilized to reduce the complexity of the noise whitening filter. The best option should be chosen according to applicable system characteristics.

The principles of noise whitening may be applied to any of the above-mentioned estimators. Depending on the data model, matrix B, that has been assumed, the estimators 100, 200, 300, 300A, and 300B may include colored-noise compensation. For example, by substituting the data model from equation (3) into equation (1), an estimator similar to that of estimator 100 is obtained, except that the estimator is now colored noise compensated. Substituting the data model from equation (11) into equation (1) yields a noise-whitened estimator similar to that of estimator 300. All of the estimators 100, 200, 300, 300A, and 300B may be altered, by changing the noise model used, to yield colored-noise-compensated estimators.

Referring again to FIGURE 4, the incoming signal ϕ is passed to each of a first FIR filter 404 and a second FIR filter 406 in order to be low-pass filtered. The first and second FIR filters 404, 406 operate in a manner similar to those of FIGURE 3B, except that $A = B^T C^{-1}$. The FIR filters 404, 406 implicitly whiten the noise based on the values of the matrix A. An output p of the first FIR filter 404 and an output r of the second FIR filter 406 are similar to the p and r values of the estimator 300B, except for the addition of the noise whitening. The outputs p and r are utilized in further calculations made in a post processor 420.

An initialization unit 422 of the estimator 400 receives the training sequence $\{b_1 \ldots b_n\}$. Depending on the assumed data model, parameter ε may also be required (see equations (13) and (14)). Once the matrix B, which represents the data model, is calculated, then matrix $B^T C^{-1} B$ is calculated. The matrix $B^T C^{-1} B$, along with the outputs p and r, is

passed to the post processor 420 as an initialized training sequence and estimates the modulation index h and the frequency offset f in accordance with the above equations.

While exemplary embodiments of the present invention have been described, it should be recognized that the invention can be varied in many ways without departing therefrom. Although the present invention has been described primarily as being used in, for example, an ad hoc wireless system operating according to BLUETOOTH, embodiments of the invention can also be used in other systems that utilize CPM. Because the invention can be varied in numerous ways, it should be understood that the invention should be limited only insofar as is required by the scope of the following claims.